Combining Stochastic Constraint Optimization and Probabilistic Programming

From Knowledge Compilation to Constraint Solving

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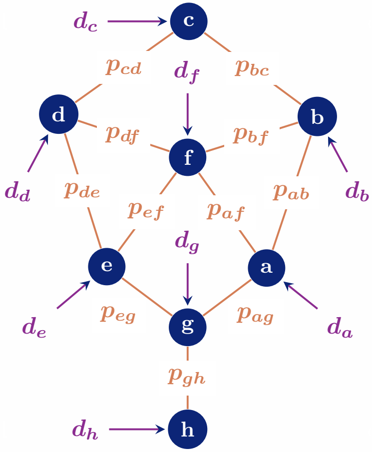
Examples of Stochastic Constraint Optimization Problems

Example problem: Viral Marketing



Targeting budget = θ

Decision to target person i directly = $d_i \in \{0, 1\}$

Expected number of people buying = $\mathbb{E}$

SCOP: **who** do we **target directly** such that $\mathbb{E}$ is **maximized** and $\sum_i d_i \leq \theta$?

D. Kempe, J. Kleinberg, É. Tardos. *"Maximizing the Spread of Influence Through a Social Network."* ACM KDD 2003.

Example problem: Theory Compression

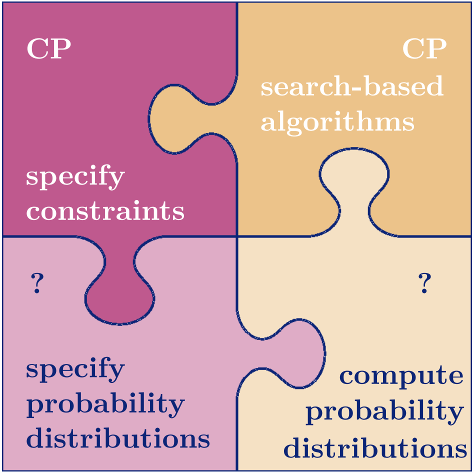
Ourfali et al., *"SPINE: a framework for signaling-regulatory pathway inference from cause-effect experiments."* Bioinformatics, 2007

Using CP to solve Stochastic Constraint Optimization Problems

Why use CP for these problems?

- They are **discrete constraint optimization** problems
- They represent a whole **class** of **similar problems**, obtainable by changing constraints and optimization criteria
- CP allows separations of the **modeling** and **solving** of these problems.

General SCOP solving method



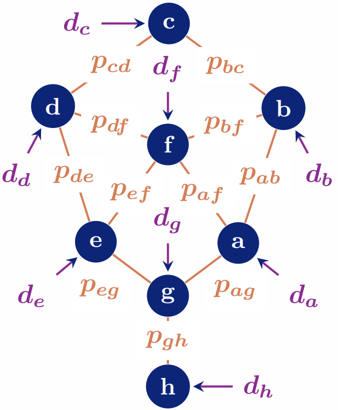
Probabilistic Logic Programming

(modeling)

L. De Raedt et al. *"ProbLog: A probabilistic Prolog and its application in link discovery." IJCAI, 2007*

G. Van den Broeck et al. *"DTProbLog: A decision-theoretic probabilistic Prolog."* AAAI, 2010

Modeling Viral Marketing with ProbLog

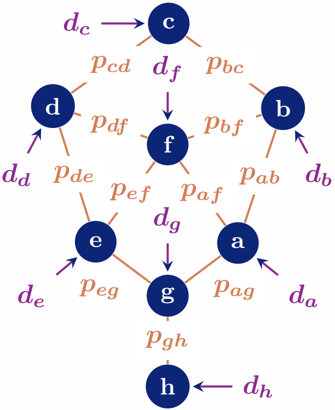



```
% Background knowledge
person(a). person(c).
person(b). person(d).   ...

% Probabilistic facts
0.7::directed(a,b).
0.4::directed(d,f).   ...

% Decision variables
?::marketed(P) :- person(P).

% Relations
trusts(X,Y) :- directed(X,Y).
trusts(Y,X) :- directed(X,Y).
buys(X) :- marketed(X).
buys(X) :- trusts(X,Y), buys(Y).

% Queries
query(buys(a)). query(buys(c)).
query(buys(b)). query(buys(d)).   ...
```

Stochastic Constraint

Probabilistic Logic Programming

CP + ProbLog = SC-ProbLog




```
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trusts(X,Y) :- directed(X,Y).
trusts(Y,X) :- directed(X,Y).
buys(X) :- marketed(X).
buys(X) :- trusts(X,Y), buys(Y).

% SCOP
{marketed(P) => 1 :- person(P).} 4.
#maximize{buys(P) => 1 :- person(P).}
```

CONTRIBUTION 1

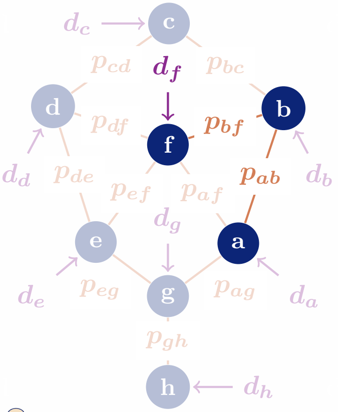
Solving with SC-ProbLog

Part A: computing probabilities
Part B: solving SCOPs the naive way
Part C: solving SCOPs with CP

L. De Raedt et al. *"ProbLog: A probabilistic Prolog and its application in link discovery." IJCAI, 2007*

G. Van den Broeck et al. *"DTProbLog: A decision-theoretic probabilistic Prolog."* AAAI, 2010

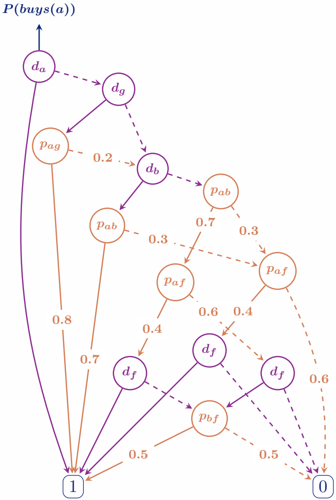
Part A: Ground program for each query

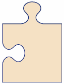


$$
\begin{aligned}
&P(buys(a)) = \\
&P(d_a \lor \\
&\quad (d_b \land p_{ab}) \lor \\
&\quad (d_f \land p_{af}) \lor \\
&\quad (d_g \land p_{ag}) \lor \\
&\quad (d_b \land p_{bf} \land p_{af}) \lor \\
&\quad (d_f \land p_{bf} \land p_{ab}) \lor \ldots)
\end{aligned}
$$

Part A: Compile to Decision Diagram

$$
\begin{aligned}
&P(buys(a)) = \\
&P(d_a \lor \\
&\quad (d_b \land p_{ab}) \lor \\
&\quad (d_f \land p_{af}) \lor \\
&\quad (d_g \land p_{ag}) \lor \\
&\quad (d_b \land p_{bf} \land p_{af}) \lor \\
&\quad (d_f \land p_{bf} \land p_{ab}) \lor \ldots)
\end{aligned}
$$

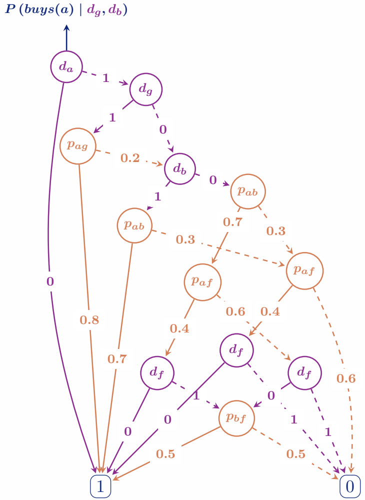




Part A: enumerate all strategies

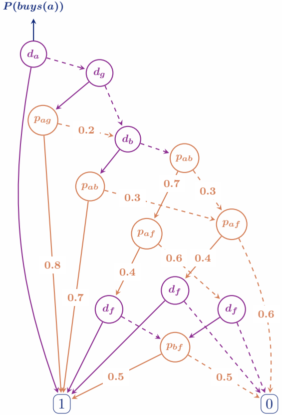
Weighted Model Counting:

$$
\begin{aligned}
&\boldsymbol{P(buys(a) \mid d_g, d_b)}\\
&\boldsymbol{P}(\bot \lor\\
&\quad (\top \land p_{ab}) \lor\\
&\quad (\bot \land p_{af}) \lor\\
&\quad (\top \land p_{ag}) \lor\\
&\quad (\top \land p_{bf} \land p_{af}) \lor\\
&\quad (\bot \land p_{bf} \land p_{ab}) \lor \ldots)
\end{aligned}
$$

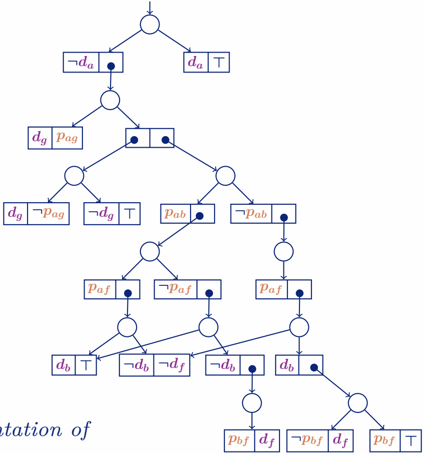


Part A: Sentential Decision Diagrams (SDDs)

Binary Decision Diagram



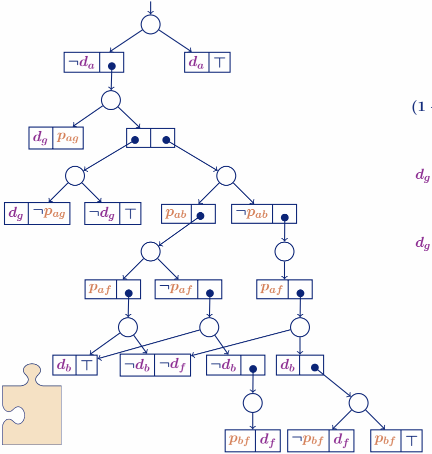
Sentential Decision Diagram



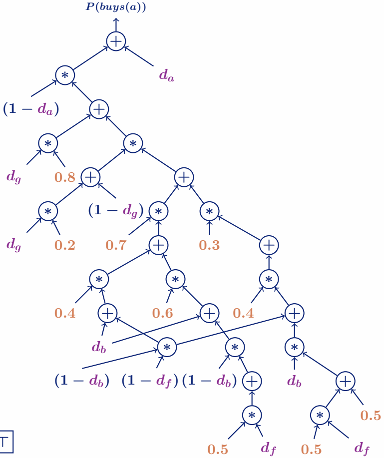
A. Darwiche. *“SDD: A New Canonical Representation of Propositional Knowledge Bases.”* AAAI 2011.

Part A: From SDD to Arithmetic Circuit

Sentential Decision Diagram



Arithmetic Circuit



Part B: Naive Solving

For each strategy $\boldsymbol{\sigma}$, the objective value for the Viral Marketing problem evaluates to:

$$\sum_{i} P(buys(i) \mid \sigma)$$

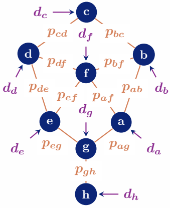
and the constraint is

$$|\sigma| \leq \theta$$

Simply **enumerate** and evaluate **all strategies** to solve the problem

Remark: ProbLog does not support this

Part B: Naive method summary



model

```
% Background knowledge
person(a). person(c).
person(b). person(d). ...

% Probabilistic facts
0.7::directed(a,b).
0.4::directed(d,f). ...

% Decision variables
?::marketed(P) :- person(P).

% Relations
trusts(X,Y) :- directed(X,Y).
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buys(X) :- trusts(X,Y), buys(Y).

% Queries
query(buys(a)). query(buys(c)).
query(buys(b)). query(buys(d)). ...
```
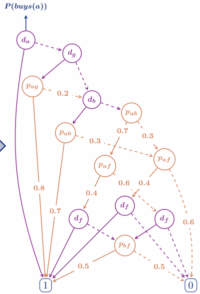

ground

(for each query $\boldsymbol{q_i}$)



enumerate

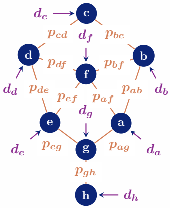
For each strategy $\boldsymbol{\sigma}$

compute $\boldsymbol{P(q_i \mid \sigma)}$ for each $\boldsymbol{q_i}$

and evaluate $\sum_i \boldsymbol{P(q_i \mid \sigma)}$

if $|\boldsymbol{\sigma}| \leq \boldsymbol{\theta}$

Part C: CP + ProbLog = SC-ProbLog



model

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% Background knowledge
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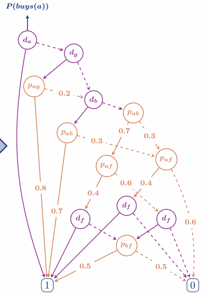
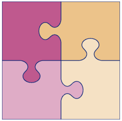
% Queries
query(buys(a)). query(buys(c)).
query(buys(b)). query(buys(d)). ...
```

ground

(for each query q_i)





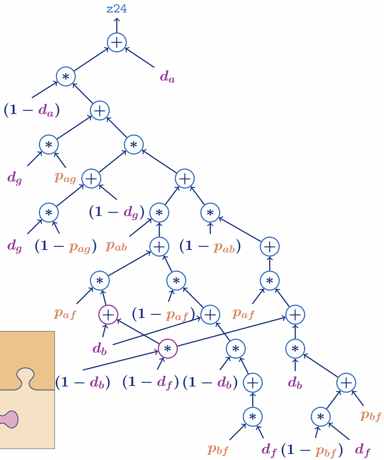
encode

Mixed-Integer Problem

Solve with CP/MIP solver

Part C: AC to Mixed Integer Problem

Arithmetic Circuit



MIP

```
% Objective Function:
maximize(z24 + ... )

% Constraint:
da + ... + dg <= theta

z24 = z23 + da
z23 = (1-da) * z22
z22 = z19 * z20
z21 = dg * pag
z20 = z14 + z15
z19 = z16 + (1-dg)
z18 = (1-pab) * z15
z17 = pab * z14
z16 = dg * (1-pag)
z15 = z13
z14 = z11 + z12
z13 = paf * z10

z12 = (1-paf) * z9
z11 = paf * z8
z10 = z5 + z7
z9 = db + z6
z8 = db + z5
z7 = (1-db) * z4
z6 = (1-db) * z3
z5 = (1-db) * (1-df)
z4 = z2 + pbf
z3 = z1
z2 = (1-pbf) * df
z1 = pbf * df
```

And solve with off-the-shelf solver…

Part C: Linearizability of SDDs

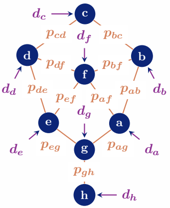
Typically, SDDs yield **quadratic** MIPs
SDDs with **special property** yield **linear** MIPs

Smaller SDDs yield smaller MIPs
Minimization typically **destroys** the special property in SDDs that makes them **linear**

Solution: custom minimization algorithm that **preserves linearity**

A. Choi, A. Darwiche. *"Dynamic Minimization of Sentential Decision Diagrams."* AAAI 2013.

Part C: SC-ProbLog Summary



model

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buys(X) :- trusts(X,Y), buys(Y).

% Queries
query(buys(a)). query(buys(c)).
query(buys(b)). query(buys(d)). ...
```
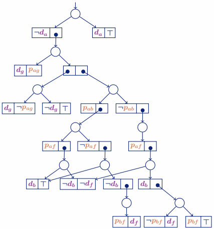
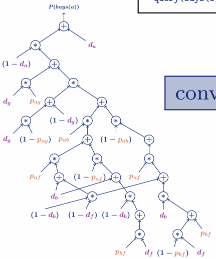

ground

(for each query q_i)



get AC

(for each query q_i)

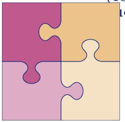


convert

```
% Objective Function:
maximize(z24 + ... )

% Constraint:
da + ... + dg <= theta

z24 = z23 + da
z23 = (1-da) * z22
z22 = z19 * z20
z21 = dg * pag
z20 = z14 + z15
z19 = z16 + (1-dg)
z18 = (1-pab) * z15
z17 = pab * z14
z16 = dg * (1-pag)
z15 = z13
z14 = z11 + z12
z13 = paf * z10
z12 = (1-paf) * z9
z11 = paf * z8
z10 = z5 + z7
z9 = db + z6
z8 = db + z5
z7 = (1-db) * z4
z6 = (1-db) * z3
z5 = (1-db) * (1-df)
z4 = z2 + pbf
z3 = z1
z2 = (1-pbf) * df
z1 = pbf * df
```

SOLVE

(with off-the-shelf CP solver)



Experiments & results

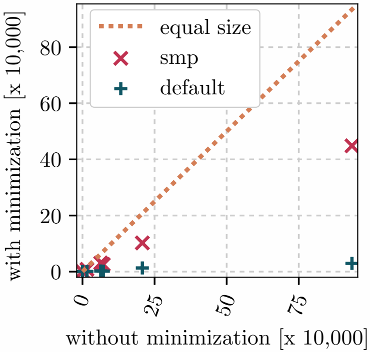
Experiments

For the experiments, we evaluate performance for:

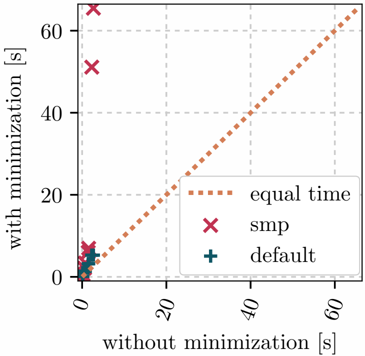
- **Sentential Decision Diagrams** (SDDs) **only**, no Ordered Binary Decision Diagrams (OBDDs)
- `Gurobi` (MIP solver) and `Gecode` (CP solver)
- **custom** minimization algorithm and **default** minimization algorithm

Results I

How do the **sizes** of the SDDs obtained compare to each other?



How does minimization influence **compilation time**?



Results II

When looking at the **total solving time** (in seconds), what is the best strategy?

instance		*size*		Gurobi		Gecode	
		n_d	n_q	no mini	smp mini	no mini	default mini
viral marketing	**setting 1**	20	20	545.8	412.7	t/o	**130.9**
viral marketing	**setting 2**	20	20	188.6	163.8	2859.9	**6.9**
viral marketing	**setting 1**	33	10	2076.8	**1185.7**	t/o	t/o
viral marketing	**setting 2**	33	10	364.6	**346.4**	t/o	t/o
theory compression	**setting 1**	36	23	3.9	**3.4**	1389.5	591.4
theory compression	**setting 2**	36	23	4.1	**3.9**	70.9	31.4
theory compression	**setting 1**	76	13	5.9	**5.6**	t/o	t/o
theory compression	**setting 2**	76	13	**4.7**	5.7	t/o	1878.2
theory compression	**setting 3**	86	26	**443.2**	471.3	t/o	t/o
theory compression	**setting 4**	71	13	23.3	21.9	222.9	**8.6**

Contributions

1. Extension of ProbLog to **SC-ProbLog**
2. **Custom SDD minimization** algorithm for producing linear MIPs
3. Proposal and implementation **SCOP solving toolchain**

Conclusion & Future work

While results are encouraging, it remains a **challenge** to solve these SCOPs on **larger networks**.

We believe that our **custom SDD minimization** algorithm can also be **applied** in **other contexts**.

Interested? Find **code** at

`https://bitbucket.org/antondries/problog/branch/sc-problog`,

or **e-mail** us at

`a.l.d.latour@liacs.leidenuniv.nl`

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We thank **Luc De Raedt** for his support, for his advice and for the numerous other ways in which he contributed to this work.

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Theme by Joost Schalken. Updated by Pepijn van Heiningen & Anna Latour.