Solving the Identifying Code Set Problem with Grouped Independent Support

Anna L.D. Latour[1], Arunabha Sen[2] and Kuldeep S. Meel[1]

[1]School of Computing, National University of Singapore

[2]Computer Science and Engineering Faculty, Arizona State University

IJCAI, 23 August 2023, Macau

Motivation

Problem

There will always be a problem whose encoding is too big.

Motivation

Problem

There will always be a problem whose encoding is too big.

Solution

Sacrifice some desiderata (*e.g.*, theoretical guarantees).

Motivation

Problem

There will always be a problem whose encoding is too big.

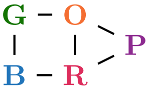
Solution

Sacrifice some desiderata (*e.g.*, theoretical guarantees).

Question

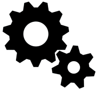
Which trade-offs can we make for an exponentially more succinct encoding?

Main contributions

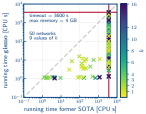


A **case study** that reduces the NP-hard **generalised identifying code set (GICS)** problem to the computationally harder **GIS** problem.

An extension of the independent support of a Boolean formula: **Grouped Independent Support (GIS)**.

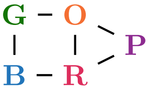


A new solver, **gismo**, for finding a grouped independent support.



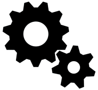
Experiments that **demonstrate the effectiveness** of reducing GICS to GIS and solving with **gismo**.

Main contributions

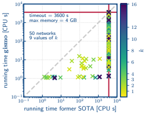


A **case study** that reduces the NP-hard **generalised identifying code set (GICS)** problem to the computationally harder **GIS** problem.

An extension of the independent support of a Boolean formula: **Grouped Independent Support (GIS)**.

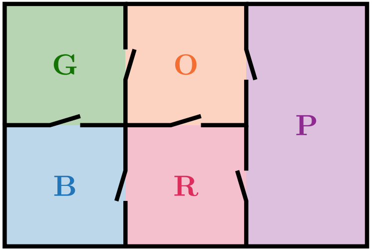


A new solver, **gismo**, for finding a grouped independent support.



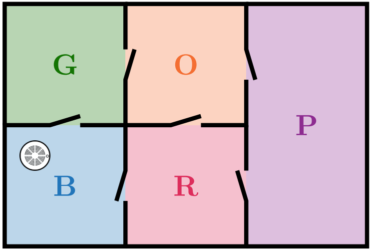
Experiments that **demonstrate the effectiveness** of reducing GICS to GIS and solving with **gismo**.

Problem (1/3)



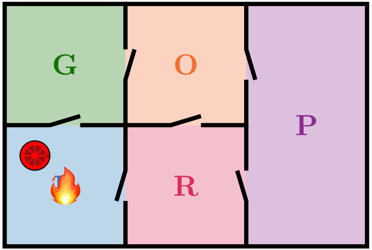
Problem (1/3)

	signature	
fire	t_0	t_1

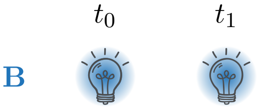


Problem (1/3)

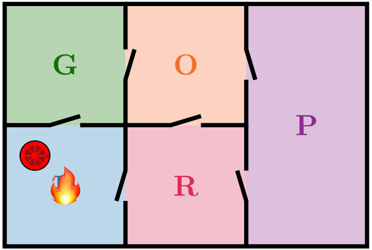
	signature	
fire	t_0	t_1
{B}	{B}	



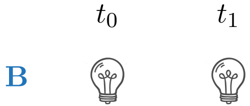
Problem (1/3)



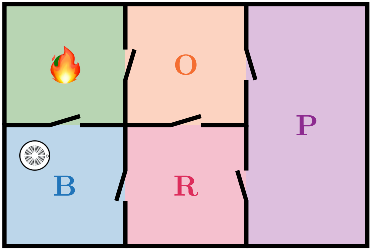
	signature	
fire	t_0	t_1
$\{B\}$	$\{B\}$	$\{B\}$



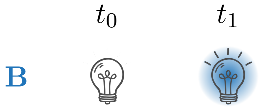
Problem (1/3)



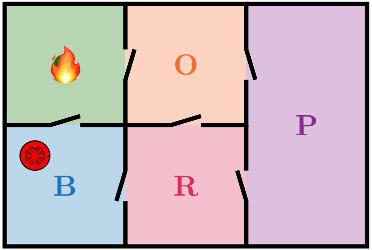
	signature	
fire	t_0	t_1
{B}	{B}	{B}
{G}	$\varnothing$	



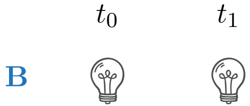
Problem (1/3)



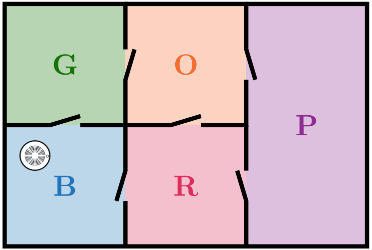
	signature	
fire	t_0	t_1
$\{B\}$	$\{B\}$	$\{B\}$
$\{G\}$	$\varnothing$	$\{B\}$



Problem (1/3)

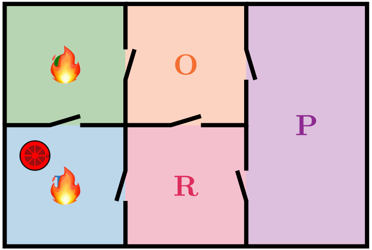


fire	signature t_0	signature t_1
{B}	{B}	{B}
{G}	$\varnothing$	{B}
{O}	$\varnothing$	$\varnothing$
{R}	$\varnothing$	{B}
{P}	$\varnothing$	$\varnothing$

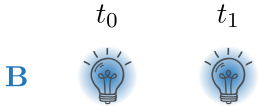


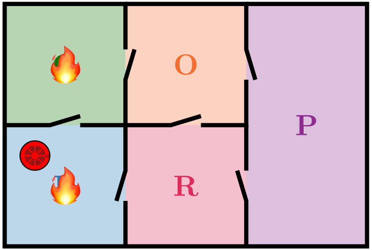
Problem (1/3)

fire	signature t_0	signature t_1
{B}	{B}	{B}
{G}	∅	{B}
{O}	∅	∅
{R}	∅	{B}
{P}	∅	∅
{B, G}	{B}	



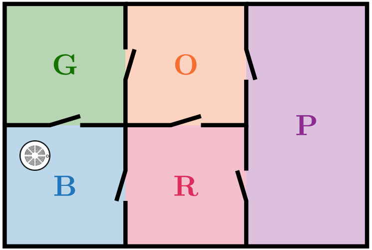
Problem (1/3)





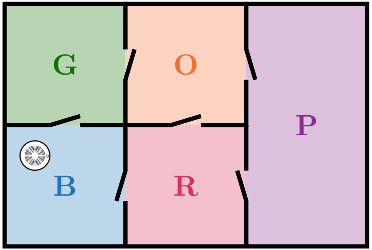
	signature	
fire	t_0	t_1
{B}	{B}	{B}
{G}	∅	{B}
{O}	∅	∅
{R}	∅	{B}
{P}	∅	∅
{B, G}	{B}	{B}

Problem (1/3)



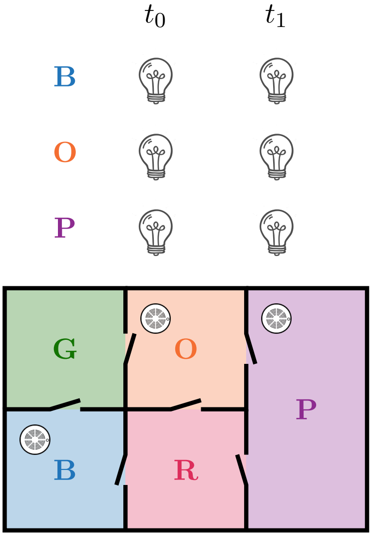
	signature	
fire	t_0	t_1
$\{B\}$	$\{B\}$	$\{B\}$
$\{G\}$	$\varnothing$	$\{B\}$
$\{O\}$	$\varnothing$	$\varnothing$
$\{R\}$	$\varnothing$	$\{B\}$
$\{P\}$	$\varnothing$	$\varnothing$
$\{B, G\}$	$\{B\}$	$\{B\}$
$\{B, O\}$	$\{B\}$	$\{B\}$
$\vdots$	$\vdots$	$\vdots$
$\{O, R, P\}$	$\varnothing$	$\{B\}$
$\vdots$	$\vdots$	$\vdots$

Problem (1/3)



	signature	
fire	t_0	t_1
{B}	{B}	{B}
{G}	∅	{B}
{O}	∅	∅
{R}	∅	{B}
{P}	∅	∅
{B, G}	{B}	{B}
{B, O}	{B}	{B}
⋮	⋮	⋮
{O, R, P}	∅	{B}
⋮	⋮	⋮
∅	∅	∅

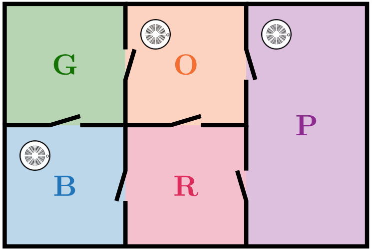
Problem (2/3)



	signature	
fire	t_0	t_1
{B}	{B}	{B}
{G}	$\varnothing$	{B, O}
{O}	{O}	{O, P}
{R}	$\varnothing$	{B, O, P}
{P}	{P}	{O, P}
{B, G}	{B}	{B, O}
{B, O}	{B, O}	{B, O}
$\vdots$	$\vdots$	$\vdots$
{O, R, P}	{O, P}	{B, O, P}
$\vdots$	$\vdots$	$\vdots$
$\varnothing$	$\varnothing$	$\varnothing$

Problem (3/3)

The set of rooms with a detector, D, is called a **generalised identifying code set (GICS)** (Karpovsky, Chakrabarty, and Levitin 1998) for positive integer k if each set of at most k fires has a unique signature.

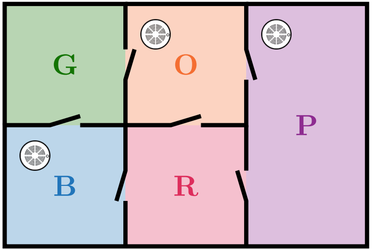


	signature	
fire	t_0	t_1
$\{B\}$	$\{B\}$	$\{B\}$
$\{G\}$	$\varnothing$	$\{B, O\}$
$\{O\}$	$\{O\}$	$\{O, P\}$
$\{R\}$	$\varnothing$	$\{B, O, P\}$
$\{P\}$	$\{P\}$	$\{O, P\}$
$\{B, G\}$	$\{B\}$	$\{B, O\}$
$\{B, O\}$	$\{B, O\}$	$\{B, O\}$
$\vdots$	$\vdots$	$\vdots$
$\{O, R, P\}$	$\{O, P\}$	$\{B, O, P\}$
$\vdots$	$\vdots$	$\vdots$
$\varnothing$	$\varnothing$	$\varnothing$

Problem (3/3)

The set of rooms with a detector, D, is called a **generalised identifying code set (GICS)** (Karpovsky, Chakrabarty, and Levitin 1998) for positive integer k if each set of at most k fires has a unique signature.

Problem: minimise $|D|$

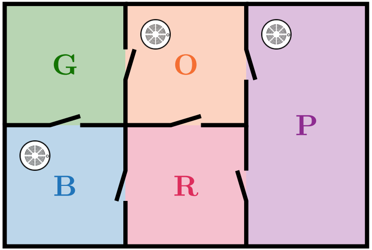


	signature	
fire	t_0	t_1
$\{B\}$	$\{B\}$	$\{B\}$
$\{G\}$	$\varnothing$	$\{B, O\}$
$\{O\}$	$\{O\}$	$\{O, P\}$
$\{R\}$	$\varnothing$	$\{B, O, P\}$
$\{P\}$	$\{P\}$	$\{O, P\}$
$\{B, G\}$	$\{B\}$	$\{B, O\}$
$\{B, O\}$	$\{B, O\}$	$\{B, O\}$
$\vdots$	$\vdots$	$\vdots$
$\{O, R, P\}$	$\{O, P\}$	$\{B, O, P\}$
$\vdots$	$\vdots$	$\vdots$
$\varnothing$	$\varnothing$	$\varnothing$

Problem (3/3)

The set of rooms with a detector, D, is called a **generalised identifying code set (GICS)** (Karpovsky, Chakrabarty, and Levitin 1998) for positive integer k if each set of at most k fires has a unique signature.

Problem: minimise $|D|$



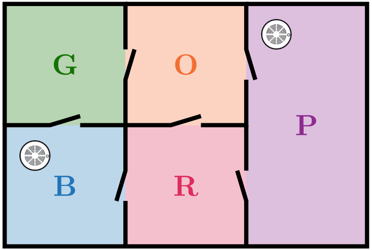
fire	signature t_0	t_1
$\{B\}$	$\{B\}$	$\{B\}$
$\{G\}$	$\varnothing$	$\{B, O\}$
$\{O\}$	$\{O\}$	$\{O, P\}$
$\{R\}$	$\varnothing$	$\{B, O, P\}$
$\{P\}$	$\{P\}$	$\{O, P\}$
$\varnothing$	$\varnothing$	$\varnothing$

Example:
$k = 1,\ D = \{B, O, P\}$

Problem (3/3)

The set of rooms with a detector, D, is called a **generalised identifying code set (GICS)** (Karpovsky, Chakrabarty, and Levitin 1998) for positive integer k if each set of at most k fires has a unique signature.

Problem: minimise $|D|$



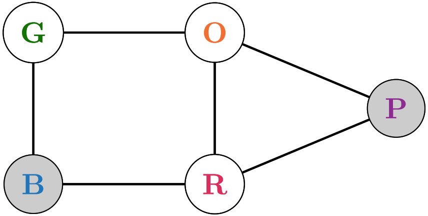
	signature	
fire	t_0	t_1
$\{B\}$	$\{B\}$	$\{B\}$
$\{G\}$	$\varnothing$	$\{B\}$
$\{O\}$	$\varnothing$	$\{P\}$
$\{R\}$	$\varnothing$	$\{B, P\}$
$\{P\}$	$\{P\}$	$\{P\}$
$\varnothing$	$\varnothing$	$\varnothing$

Example:
$k = 1,\ D = \{B, P\}$

Problem (3/3)

The set of rooms with a detector, D, is called a **generalised identifying code set (GICS)** (Karpovsky, Chakrabarty, and Levitin 1998) for positive integer k if each set of at most k fires has a unique signature.

Problem: minimise $|D|$



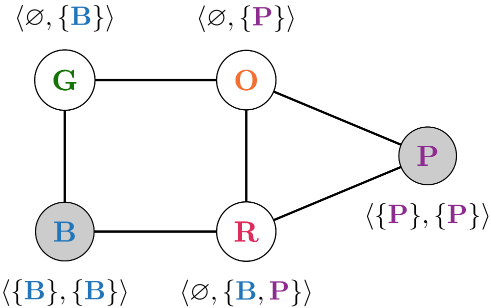
	signature	
fire	t_0	t_1
$\{\mathbf{B}\}$	$\{\mathbf{B}\}$	$\{\mathbf{B}\}$
$\{\mathbf{G}\}$	$\varnothing$	$\{\mathbf{B}\}$
$\{\mathbf{O}\}$	$\varnothing$	$\{\mathbf{P}\}$
$\{\mathbf{R}\}$	$\varnothing$	$\{\mathbf{B}, \mathbf{P}\}$
$\{\mathbf{P}\}$	$\{\mathbf{P}\}$	$\{\mathbf{P}\}$
$\varnothing$	$\varnothing$	$\varnothing$

Example:
$k = 1,\ D = \{\mathbf{B}, \mathbf{P}\}$

Problem (3/3)

The set of rooms with a detector, D, is called a **generalised identifying code set (GICS)** (Karpovsky, Chakrabarty, and Levitin 1998) for positive integer k if each set of at most k fires has a unique signature.

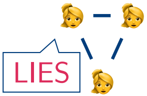
Problem: minimise $|D|$



	signature	
fire	t_0	t_1
$\{B\}$	$\{B\}$	$\{B\}$
$\{G\}$	$\varnothing$	$\{B\}$
$\{O\}$	$\varnothing$	$\{P\}$
$\{R\}$	$\varnothing$	$\{B, P\}$
$\{P\}$	$\{P\}$	$\{P\}$
$\varnothing$	$\varnothing$	$\varnothing$

Example:
$k = 1,\ D = \{B, P\}$

Applications of Identifying Code Sets



Identifying sources of misinformation (Basu and Sen 2021a).

Identifying criminals in social networks (Basu and Sen 2021b).

Satellite deployment (Sen, Goliber, Basu, Zhou, and Ghosh 2019).

Satellite image: Flaticon.com

Solving the GICS problem

Former state of the art

(Padhee, Biswas, Pal, Basu, and Sen 2020)

1. Encode problem in integer-linear program (ILP).
2. Solve with CPLEX.

Solving the GICS problem

Former state of the art

(Padhee, Biswas, Pal, Basu, and Sen 2020)

1. Encode problem in integer-linear program (ILP).
2. Solve with CPLEX.

- **Exponential** # constraints.

Solving the GICS problem

Former state of the art

(Padhee, Biswas, Pal, Basu, and Sen 2020)

1. Encode problem in integer-linear program (ILP).
2. Solve with CPLEX.

- **Exponential** # constraints.

New approach (contribution)

1. Reduce GICS problem to finding a minimal **grouped independent support** (GIS).

Solving the GICS problem

Former state of the art

(Padhee, Biswas, Pal, Basu, and Sen 2020)

1. Encode problem in integer-linear program (ILP).
2. Solve with CPLEX.

- **Exponential** # constraints.

New approach (contribution)

1. Reduce GICS problem to finding a minimal **grouped independent support** (GIS).

- **Linear** # clauses.

Solving the GICS problem

Former state of the art

(Padhee, Biswas, Pal, Basu, and Sen 2020)

1. Encode problem in integer-linear program (ILP).
2. Solve with CPLEX.

- **Exponential** # constraints.
- Checking if candidate is a solution: **polytime**.

New approach (contribution)

1. Reduce GICS problem to finding a minimal **grouped independent support** (GIS).

- **Linear** # clauses.
- Checking if candidate is a GIS: **co-NP**.

Solving the GICS problem

Former state of the art

(Padhee, Biswas, Pal, Basu, and Sen 2020)

1. Encode problem in integer-linear program (ILP).
2. Solve with CPLEX.

- **Exponential** # constraints.
- Checking if candidate is a solution: **polytime**.
- **Cardinality-minimal** solution D.

New approach (contribution)

1. Reduce GICS problem to finding a minimal **grouped independent support** (GIS).
2. Use **gismo** to find a GIS.

- **Linear** # clauses.
- Checking if candidate is a GIS: **co-NP**.
- **Set-minimal** solution D.

Background: Propositional Logic

Solution $\sigma : X \mapsto \{0, 1\}$ maps variables to truth values.

Example: $F(X) := (x_1 \vee x_2) \leftrightarrow x_3$

	x_1	x_2	x_3
σ_1	1	1	1
σ_2	1	0	1
σ_3	0	1	1
σ_4	0	0	0

Background: Propositional Logic

Solution $\sigma : X \mapsto \{0, 1\}$ maps variables to truth values.

Example: $F(X) := (x_1 \vee x_2) \leftrightarrow x_3$

	x_1	x_2	x_3
σ_1	1	1	1
σ_2	1	0	1
σ_3	0	1	1
σ_4	0	0	0

Projection set: $S := \{x_1, x_3\}$

Background: Propositional Logic

Solution $\sigma : X \mapsto \{0,1\}$ maps variables to truth values.

Example: $F(X) := (x_1 \vee x_2) \leftrightarrow x_3$

	x_1	x_2	x_3
σ_1	1	1	1
σ_2	1	0	1
σ_3	0	1	1
σ_4	0	0	0

Projection set: $S := \{x_1, x_3\}$

$$|Sol_{\downarrow S}(F)| \leq |Sol(F)|$$

Background: Propositional Logic

Solution $\sigma : X \mapsto \{0,1\}$ maps variables to truth values.

Example: $F(X) := (x_1 \lor x_2) \leftrightarrow x_3$

	x_1	x_2	x_3
σ_1	1	1	1
σ_2	1	0	1
σ_3	0	1	1
σ_4	0	0	0

Projection set: $S := \{x_1, x_3\}$

	x_1	x_2	x_3
σ_1	1	1	1
σ_2	1	0	1
σ_3	0	1	1
σ_4	0	0	0

Projection set: $I := \{x_1, x_2\}$

$$|Sol_{\downarrow S}(F)| \leq |Sol(F)|$$

Background: Propositional Logic

Solution $\sigma : X \mapsto \{0,1\}$ maps variables to truth values.

Example: $F(X) := (x_1 \vee x_2) \leftrightarrow x_3$

	x_1	x_2	x_3
σ_1	1	1	1
σ_2	1	0	1
σ_3	0	1	1
σ_4	0	0	0

Projection set: $S := \{x_1, x_3\}$

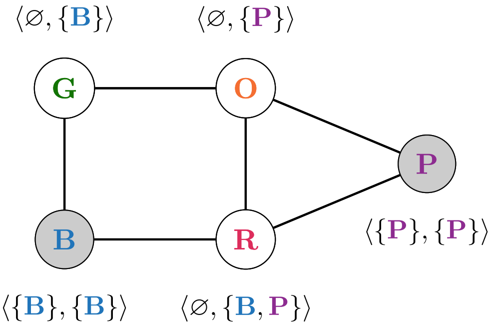
$$|Sol_{\downarrow S}(F)| \leq |Sol(F)|$$

	x_1	x_2	x_3
σ_1	1	1	1
σ_2	1	0	1
σ_3	0	1	1
σ_4	0	0	0

Projection set: $I := \{x_1, x_2\}$ is an **independent support** (Chakraborty, Fremont, Meel, Seshia, and Vardi 2014) of $F(X)$.

$$|Sol_{\downarrow I}(F)| = |Sol(F)|$$

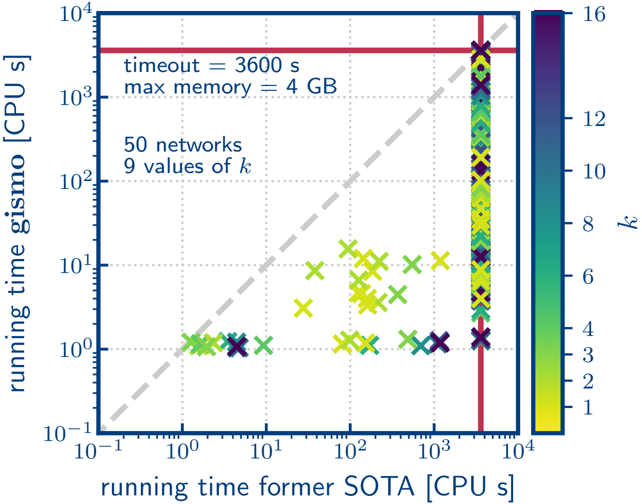
Contribution: Reduction of GICS to GIS



Our method

- Encode GICS in CNF formula
  - each solution corresponds to the signature s_U of a $U \subseteq V$ with $|U| \leq k$;
  - **linear** size.
- Two variables per group.
- One variable group for each node.
- Use **gismo** to find minimal GIS.
- Groups in GIS correspond to nodes in D.

Results



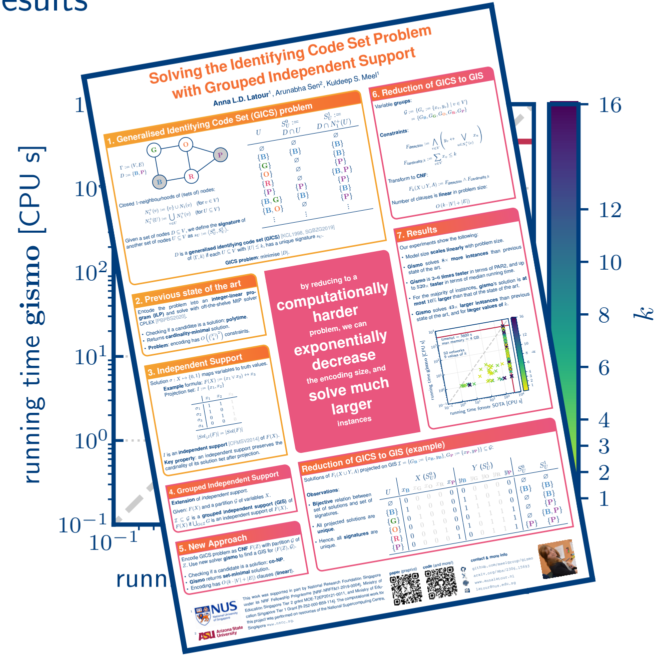
paper (preprint)

arxiv.org/abs/2306.15693

code (and more!)

github.com/meelgroup/gismo

Results



paper (preprint)

arxiv.org/abs/2306.15693

code (and more!)

github.com/meelgroup/gismo

References I

Basu, Kaustav and Arunabha Sen (2021a). "Epidemiological Model Independent Misinformation Source Identification". In: Proceedings of the ICWSM Workshops.

— (2021b). "Identifying individuals associated with organized criminal networks: A social network analysis". In: *Soc. Networks* 64, pp. 42–54.

Padhee, Malhar, Reetam Sen Biswas, Anamitra Pal, Kaustav Basu, and Arunabha Sen (2020). "Identifying Unique Power System Signatures for Determining Vulnerability of Critical Power System Assets". In: *SIGMETRICS Perform. Evaluation Rev.* 47.4, pp. 8–11.

Sen, Arunabha, Victoria Horan Goliber, Kaustav Basu, Chenyang Zhou, and Sumitava Ghosh (2019). "On upper and lower bounds of identifying code set for soccer ball graph with application to satellite deployment". In: *ICDCN*. ACM, pp. 307–316.

Chakraborty, Supratik, Daniel J. Fremont, Kuldeep S. Meel, Sanjit A. Seshia, and Moshe Y. Vardi (2014). "Distribution-Aware Sampling and Weighted Model Counting for SAT". In: *AAAI*. AAAI Press, pp. 1722–1730.

Karpovsky, Mark G., Krishnendu Chakrabarty, and Lev B. Levitin (1998). "On a New Class of Codes for Identifying Vertices in Graphs". In: *IEEE Trans. Inf. Theory* 44.2, pp. 599–611.